

Reply to “Comment on “Dispersive Bottleneck Delaying Thermalization of Turbulent Bose-Einstein Condensates” by E. Kozik”

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Krstulovic and Brachet reply: In the preceding Comment [1], Kozik raised a criticism against the bottleneck proposed in our Letter [2] to be causing a thermalization delay when dispersive effects, controlled by the coherence length ξ , are large at truncation wavenumber: k_{\max} . The late-time energy spectrum presents a front at wavenumber $k_c(t)$ propagating toward higher wavenumbers and leaving in its wake a quasi-thermalized distribution. Kozik argues that our observations agree with the relaxation scenario, developed by Svistunov [3], that involves no bottleneck and predicts $k_c(t) \sim t^{1/4}$.

Indeed, it is apparent on Fig.1 (where $k_c(t) \sim t^\alpha$ corresponds to a line of slope $(\alpha-1)/\alpha$) that four out of eleven runs (*vi*, *vii*, *viii* and *xii*) are somewhat compatible with the Svistunov prediction. However, the prediction only

prepared using the stochastic Ginzburg-Landau equation and $\xi k_{\max} \sim 6$, have slopes closer to the Svistunov prediction of -3 . Runs that saturate with $k_c/k_{\max} \sim 1$ are reaching (truncated) thermal equilibrium and are not spectrally well-converged. In contrast, runs *i-iv*, with $\xi k_{\max} \sim 24$, saturate at $k_c/k_{\max} < 0.4$ and are well-converged but the data suggests a logarithmic growth of $k_c(t)$ (vertical line on Fig.1), a behavior very different from that predicted in [3].

This discrepancy is perhaps due to the fact that Svistunov considers a two stages process: first a condensation produced by a particle-flux wave propagating to low energies and then a wave propagating from the low to high energy region. It is not absolutely clear that the initial conditions of our Letter [2], really correspond to any of the stages considered by Svistunov (see the discussion following Eq. (4.7) of [3]).

Concerning the criticism against our use of the word “bottleneck”, we believe it is related to a limitation in Svistunov theory. Indeed, it is well known that Bogoliubov’s dispersion relation $\omega_B(k) = kc(1 + k^2\xi^2/2)^{1/2}$ (where c is the sound velocity) implies (around wavenumber $k \sim 1/\xi$) a change from propagative to dispersive behavior. This elementary point is not completely addressed in Svistunov theory, in particular at level of the kinetic equations 3.10-3.13 of [3] and Eq.(1) of [1] [7]. Thus Svistunov’s analysis is only applicable for wavenumbers $k \gg 1/\xi$. This limitation does not allow one to appreciate the importance of ξ and to grasp that $k\xi$ (in particular ξk_{\max}) is an important dimensionless parameter in this problem leading to a crossover between different regimes (see Fig.1 and also Fig.7b-c of [4]).

In a physical BEC, k_{\max} correspond to the equipartition wavenumber k_{eq} (see [2] and Sec. IV of [4]). Sinatra and Castin [5] have shown that the slowdown of thermalization reported in [2] can be related to the behavior of the (classical) damping rate around equilibrium that reaches a maximum around $k\xi \sim 3$ and decays for $k\xi \gg 1$. They have established that, at fixed $k\xi$ well beyond its maximum, the (quantum) Beliaev-Landau damping rate approaches the classical one provided $k_B T/|\psi_0|^2 g > 200$ which could be achieved experi-

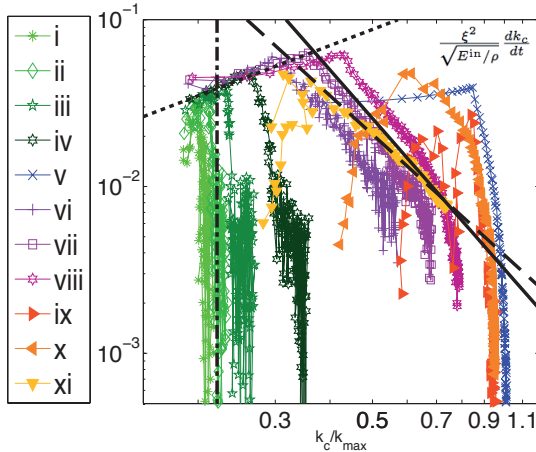


FIG. 1: (color online). Parametric representation dk_c/dt v.s. k_c/k_{\max} (adapted from Fig.4.e of [2]). Different scaling laws are displayed. Svistunov prediction: solid line $(k_c/k_{\max})^{-3}$; fit to run *xi*: large dashed line $(k_c/k_{\max})^{-2.4}$; dotted line $(k_c/k_{\max})^1$ and dotted-dashed line $(k_c/k_{\max})^{-0}$.

works in the limited range $0.4 < k_c/k_{\max} < 0.8$ where run *xi*, with Taylor-Green initial data and $\xi k_{\max} \sim 6$, yields a slope of -2.4 . Runs *vi*, *vii* and *viii*, with initial data

mentally using Feshbach resonance.

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